SN, MSW and MC

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SN

The neutrino signal emitted by supernovae contains a wealth of information about both the ν's and the SN.

As the ν's propagate through the overlying material MSW effects alter the fluxes and, thus, the detectable signal.

Barger, Marfatia & Wood, Phys. Lett. B, **498**, 53 (2001),
Dighe & Smirnov, PRD, **62**, 033007 (2000),
Fuller, Mayle, Wilson, & Schramm, ApJ, **322**, 795 (1987),
Fuller, Haxton & McLaughlin, PRD, **59**, 085005 (1999),
Loreti, Qian, Fuller & Balantekin, PRD, **52**, 6664 (1995)
Lunardini & Smirnov, JCAP, **6**, 9 (2003)
and many more.

Schirato & Fuller, astro-ph/0205390, showed that the shock can reach the $1 \leftrightarrow 3$ resonance.

The neutrino signal will be affected by the shock.

MSW

For 2 flavour mixing the Schrödinger equation in the matter basis is

$$\boxed{i\frac{d}{dx}\begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} k & i\theta' \\ -i\theta' & -k \end{pmatrix}\begin{pmatrix} a_H \\ a_L \end{pmatrix}}$$

a_H and a_L are complex coefficients, and after introducing,

$$\boxed{k_V = \frac{\delta m^2}{4E} \qquad V = \sqrt{2}G_F n_e}$$

then

$$\boxed{k = k_V \frac{\sin(2\theta_V)}{\sin(2\theta)} \qquad \tan(2\theta) = \frac{\sin(2\theta_V)}{\cos(2\theta_V) - V/2k_V}}$$

For a realistic density profiles the Schrödinger equation can be very tough to solve numerically.

The reasons are:

- a_H and a_L oscillate,
  - oscillatory numerical solutions are prone to error accumulation,
  - to avoid errors, the increments of the integration variable must be smaller than the oscillation length,
- the oscillation length, ~1/k, is much smaller than the extent of the density profile,
  - the differential equation solver has to take many (many) steps,
- we are most interested in the resonance region where *k* is a minimum,
  - the more time it takes to evolve the wavefunction per unit distance the less interesting its behaviour.

The quantity we're interested in calculating is the crossing probability P_C (or something related to it).

We could always use an approximation such as:

- Landau-Zener (or a variant),
- an expansion in powers of $\sin^2(2\theta_V)$,
- use a semi-classical treatment such as that of Balantekin and Beacom, PRD, **54**, 6323 (1996),
- use one of the exact solutions and adjust the parameters describing the potential.

Though useful and widely used these approximations usually break down (or converge slowly) for:

- multiple resonances,
- mildly/strongly non-adiabatic evolution,
- large vacuum mixing angles,
- or whenever phase effects are important.

MC

We needed a method for calculating P_C that didn't suffer these breakdowns but also did not face the numerical problems associated with the Schrödinger equation.

First we transformed from x to φ, which is simply

$$\boxed{\frac{d\varphi}{dx} = \frac{k}{\pi}}$$

and changed the basis from <u>a</u> to a new, adiabatic, basis <u>b</u>.

$$\boxed{\begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} e^{-i\pi\varphi} & 0 \\ 0 & e^{i\pi\varphi} \end{pmatrix} \begin{pmatrix} b_H \\ b_L \end{pmatrix}}$$

φ has the physical interpretation of being the number of half-phases of the adiabatic solution.

Having done that we end up with

$$\boxed{i\frac{d}{d\varphi}\begin{pmatrix} b_H \\ b_L \end{pmatrix} = \begin{pmatrix} 0 & i\Gamma e^{2i\pi\varphi} \\ -i\Gamma e^{-2i\pi\varphi} & 0 \end{pmatrix}\begin{pmatrix} b_H \\ b_L \end{pmatrix} = H(\varphi)\begin{pmatrix} b_H \\ b_L \end{pmatrix}}$$

The change in basis removes the k's from the Hamiltonian.

And the change in variable means we introduced a quantity Γ which is simply

$$\boxed{\Gamma = \pi/\gamma \qquad \gamma = k/\theta'}$$

with γ the adibaticity parameter.

The point of maximal violation of adiabaticity occurs at the minimal value of γ. When γ is small Γ is large.

We can integrate this equation

$$\boxed{\begin{pmatrix} b_H(\Phi) \\ b_L(\Phi) \end{pmatrix} = \begin{pmatrix} b_H(0) \\ b_L(0) \end{pmatrix} - i\int_0^\Phi d\varphi' \, H(\varphi') \begin{pmatrix} b_H(\varphi') \\ b_L(\varphi') \end{pmatrix}}$$

and repeated substitution gives us

$$\boxed{\begin{pmatrix} b_{H\Phi} \\ b_{L\Phi} \end{pmatrix} = \left\{ 1 + (-i)\int_0^\Phi d\varphi_1 \, H_1 + (-i)^2 \int_0^\Phi d\varphi_1 \int_0^{\varphi_1} d\varphi_2 \, H_1 \, H_2 + \ldots \right\} \begin{pmatrix} b_{H0} \\ b_{L0} \end{pmatrix}}$$

which defines a scattering matrix S(Φ)

$$\boxed{S(\Phi) = 1 + (-i)\int_0^\Phi d\varphi_1 \, H_1 + (-i)^2 \int_0^\Phi d\varphi_1 \int_0^{\varphi_1} d\varphi_2 \, H_1 \, H_2 + \ldots}$$

We can calculate S with a Monte Carlo integrator.

Our MC uses importance sampling for φ.

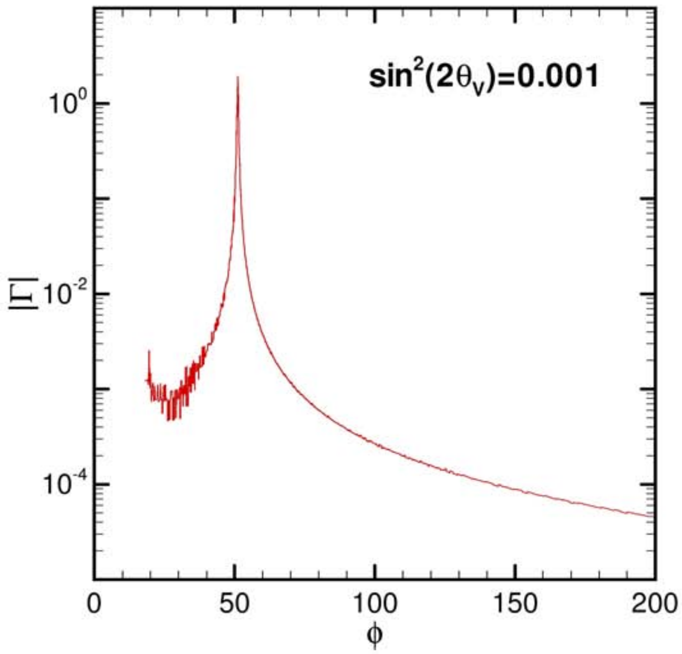
Our first guess would be to identify the probability distribution P(φ) as being proportional to Γ.

Unfortunately, because

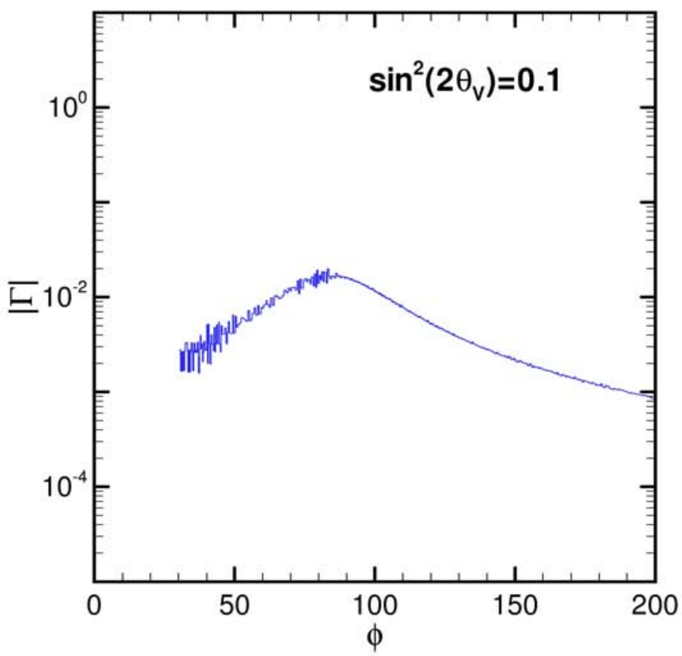
$$\boxed{\Gamma = \pi / \gamma = \pi \, \theta' / k}$$

then, for non-monotonic profiles, Γ, and so P(φ), would not be positive definite.

Instead we opt for P(φ) ∝ |Γ|.



For non-adiabatic propagation $|\Gamma|$ is narrow and our random values of φ will be cluster around the peak.



For adiabatic propagation $|\Gamma|$ is broad, our random values of φ will extend over a wide range.

Standard Solar Model profile.

$\delta m^2 = 8 \times 10^{-5}$ eV^2, E = 10 MeV

We introduce the reduced Hamiltonian h(φ) such that
H(φ) = P(φ) h(φ) with

$$\boxed{h(\varphi) = \frac{sign[\Gamma(\varphi)]}{N}\begin{pmatrix} 0 & ie^{2i\pi\varphi} \\ -ie^{-2i\pi\varphi} & 0 \end{pmatrix}}$$

Various identities allow us to alter all the upper limits to Φ

and we also use the identity $\boxed{1 = \int_0^\Phi P(\varphi)\, d\varphi}$

to collapse the sum of integrals to just one of infinite measure.

Our expression for S becomes

$$\boxed{S = \left(\prod_{i=1}^{\infty}\int_0^\Phi P(\varphi_i)\, d\varphi_i\right)\left\{1 - ih_1 + \frac{(-i)^2}{2!}\mathrm{T}(h_1 h_2) + \ldots\right\}}$$

All the MSW effects are in S. The structure of S is

$$\boxed{S = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}}$$

The crossing probability is simply

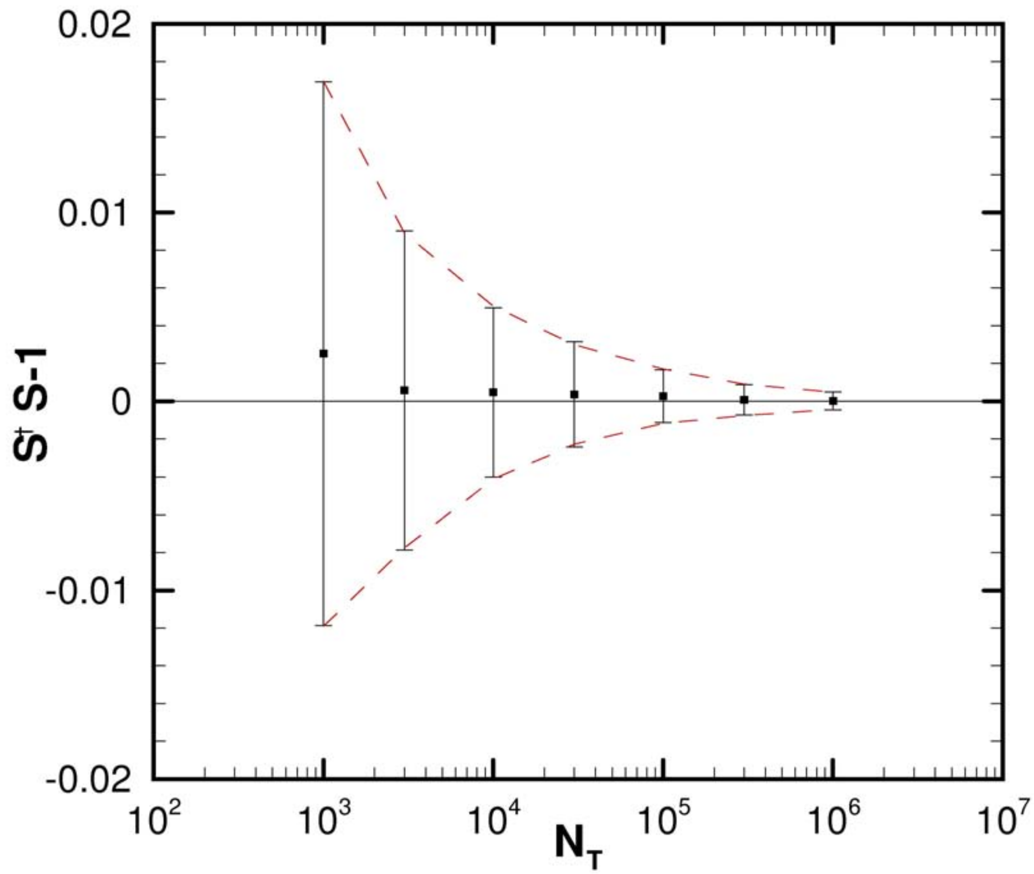
$$\boxed{1 - P_c^{(a)} = |a|^2 \qquad P_c^{(b)} = |b|^2}$$

Due to the finite sample size, S is not unitary.

$S^{\dagger}S - 1 \neq 0$

$|a|^2 + |b|^2 - 1 \neq 0$

$P_c^{(b)} - P_c^{(a)} \neq 0$

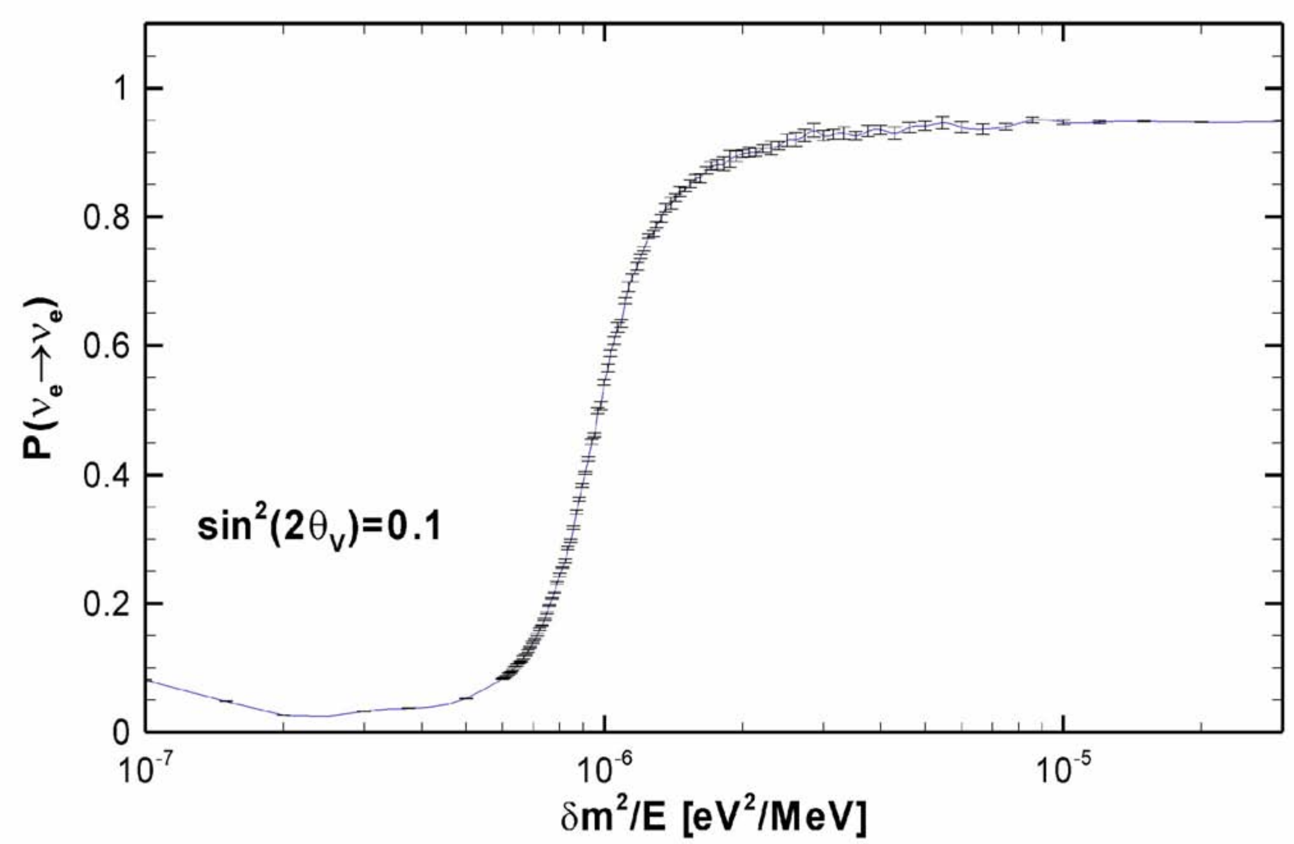


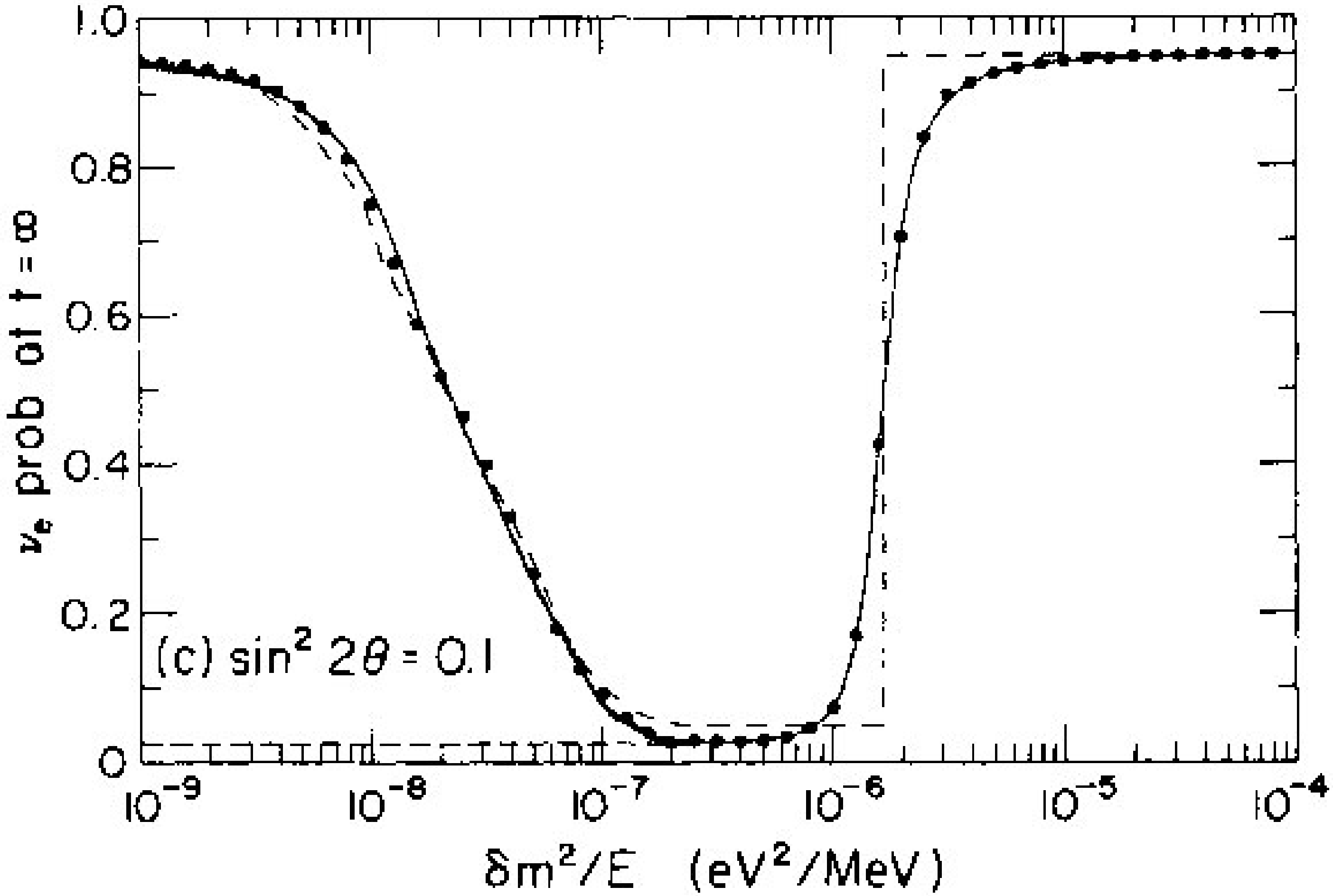
2005 Standard Solar Model profile.

$\delta m^2 = 3 \times 10^{-5}$ eV^2, $\sin^2(2\theta_V) = 0.001$,
E = 10 MeV

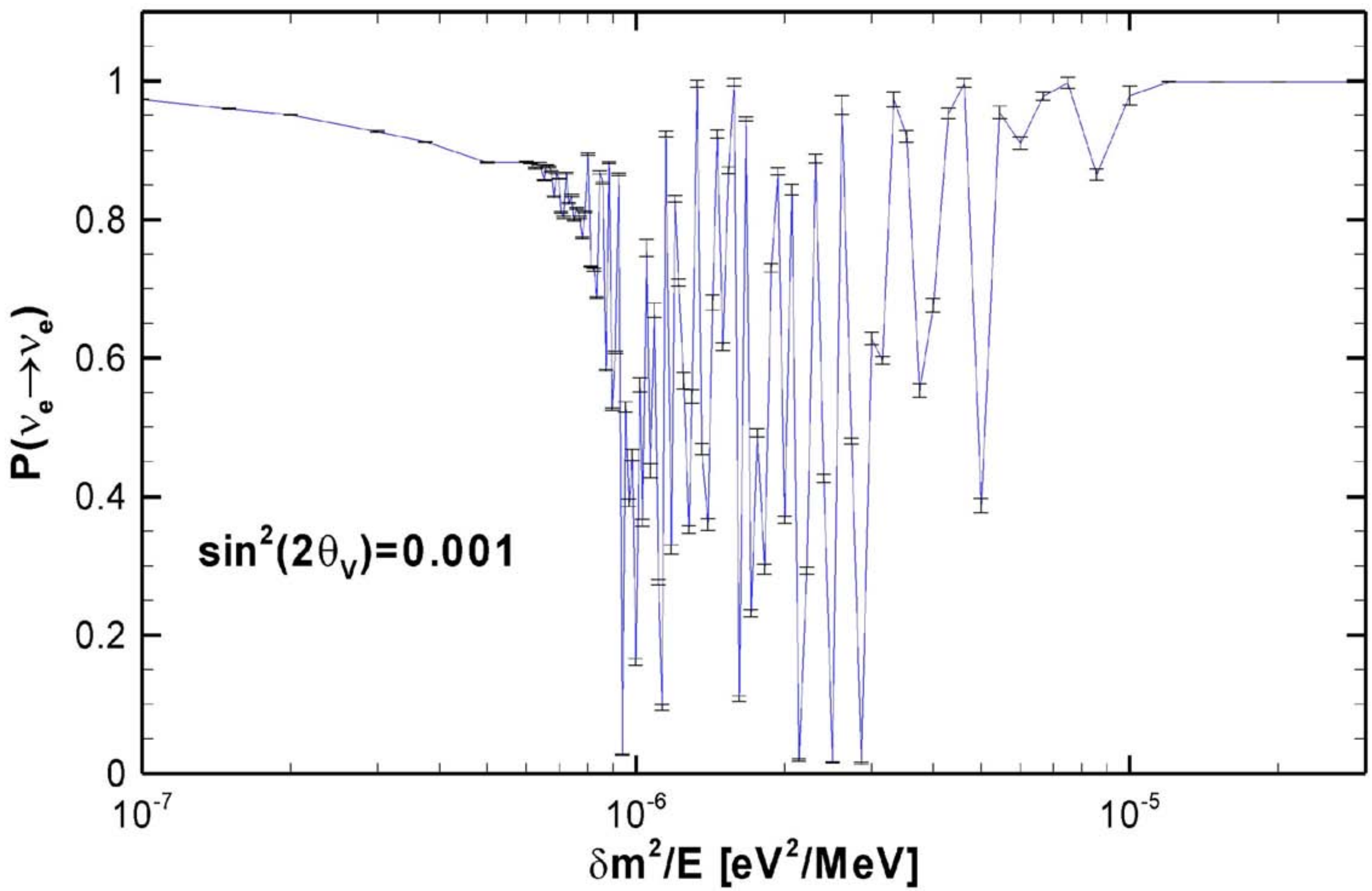
To reach an accuracy of 0.1% then $N_T \sim 10^6$.

One of the tests of the code was to calculate the electron neutrino survival probability for neutrinos produced at 0.3 $R_{\odot}$ that propagate back through the core of the Sun.

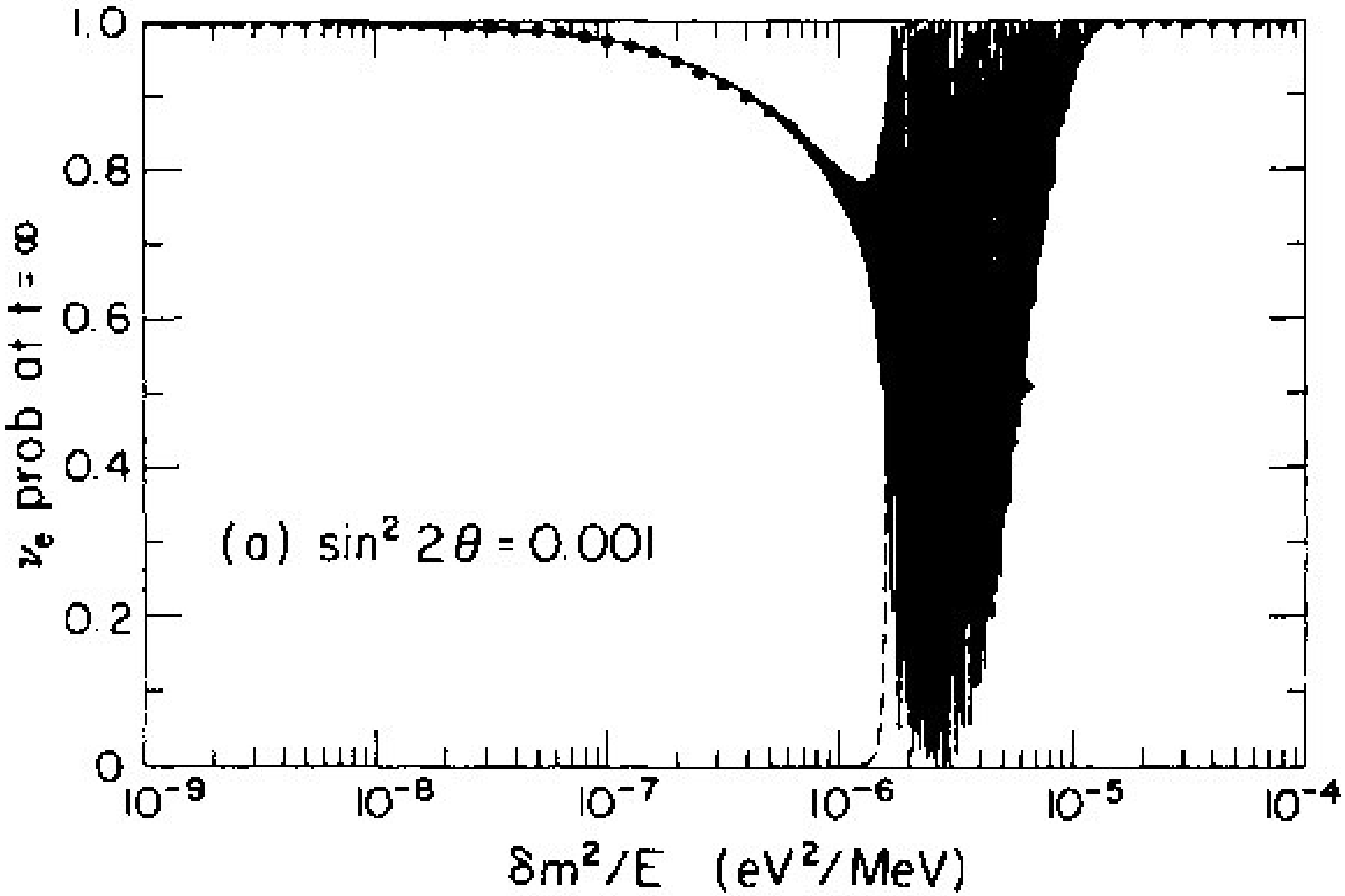




This same problem was considered by
Haxton, PRD, **35**, 2352 (1987)



Again, the neutrinos are produced at 0.3 $R_{\odot}$ and propagate back through the core of the Sun.



Haxton, PRD, **35**, 2352 (1987)

Again, the neutrinos are produced at 0.3 $R_\odot$ and propagate back through the core of the Sun.

SN

Pulsar velocities and the observation of polarized light indicate SN are aspherical.

Blondin, Mezzacappa & DeMarino, ApJ, **584**, 971 (2003) found that perturbations in the standing accretion shock can generate large $\ell = 1$, $\ell = 2$ moments.

To examine if this can seen in the neutrinos we have to:

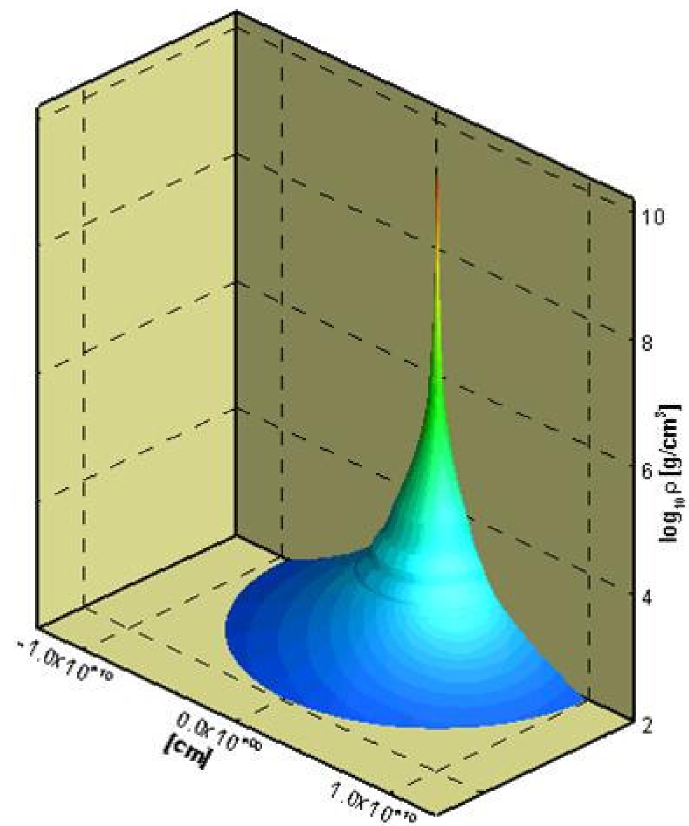
1. make an aspherical SN,
2. calculate P_C as a function of time, angle, and energy,
3. determine the effects upon the fluxes and observables.

Our 2D calculation used VH-1 and a 13.2 $M_\odot$ progenitor model from Heger (www.ucolick.org/alex/stellarevolution).

We had to help our simulations explode.

To do this we increased the radial velocities for the inner 100 km according to a $\cos^2(\theta)$ function.

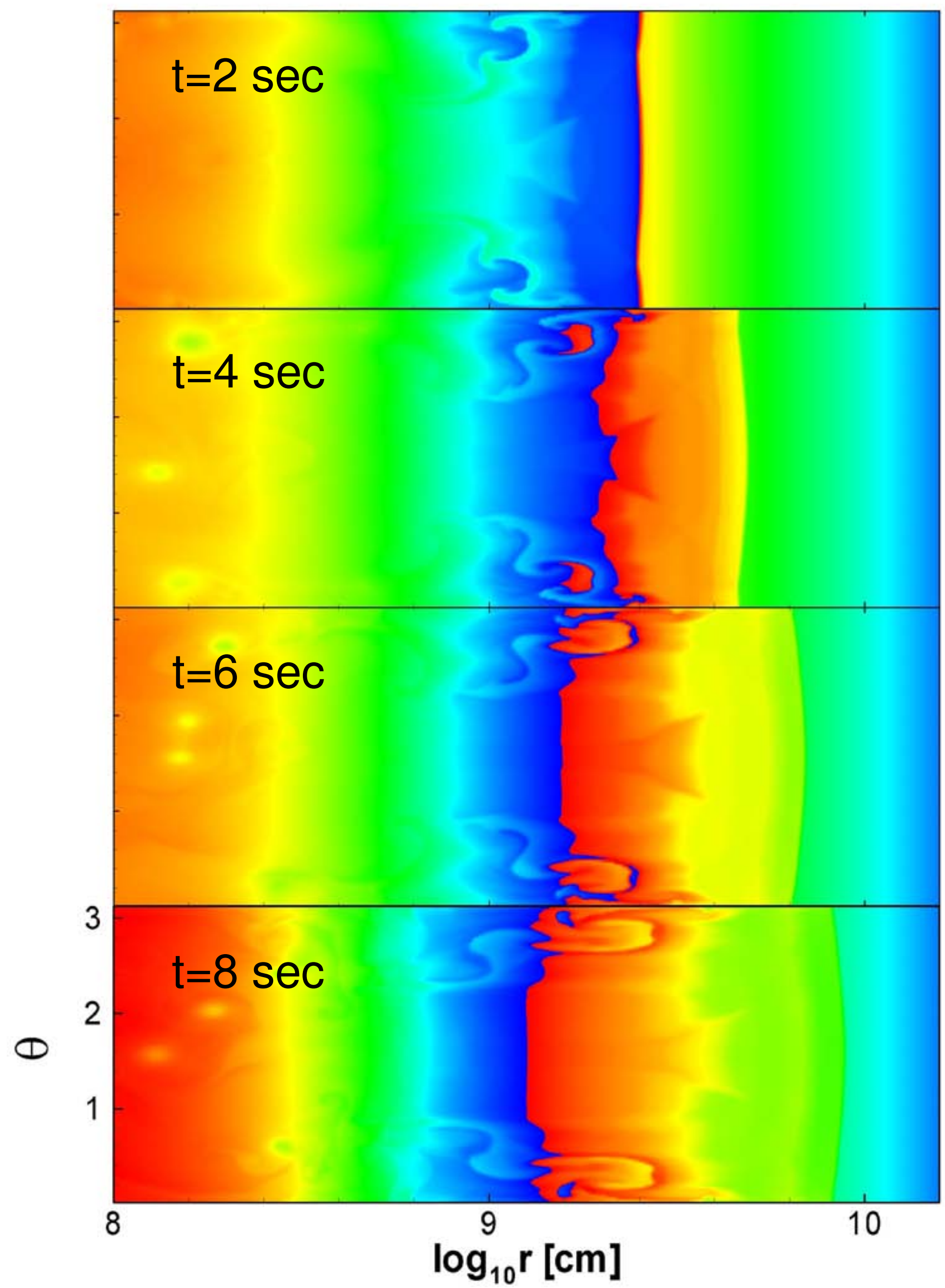
The total energy increase was approximately 10^{51} ergs.

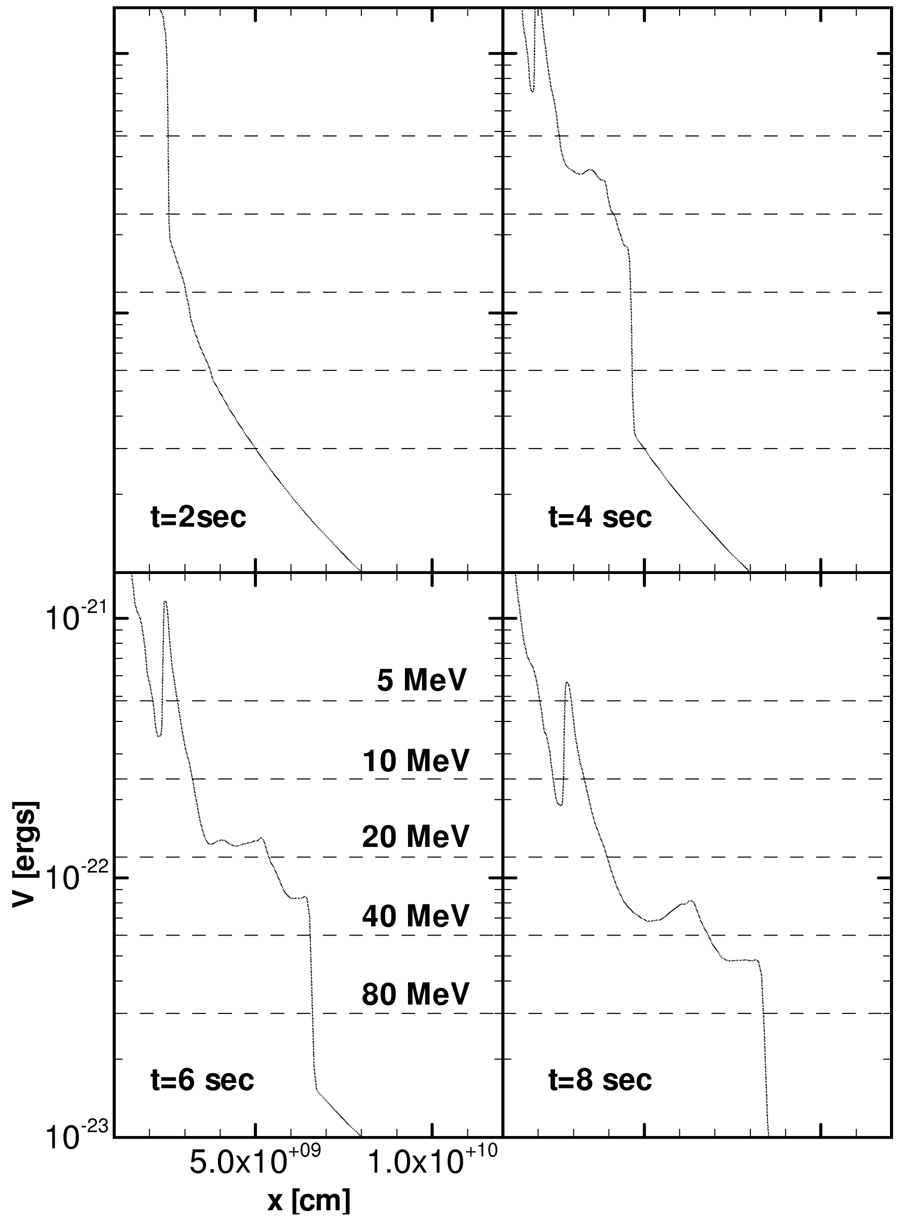


The profile varies with angle and 'bubbles' form in the region behind the shock.

In some radial directions the density is not a monotonically decreasing function of the radius.

Within some range in energy, neutrinos that pass through a bubble will experience triple resonances.





As the shock moves forward the neutrinos that experience a non-adiabtic resonance moves through the spectrum.

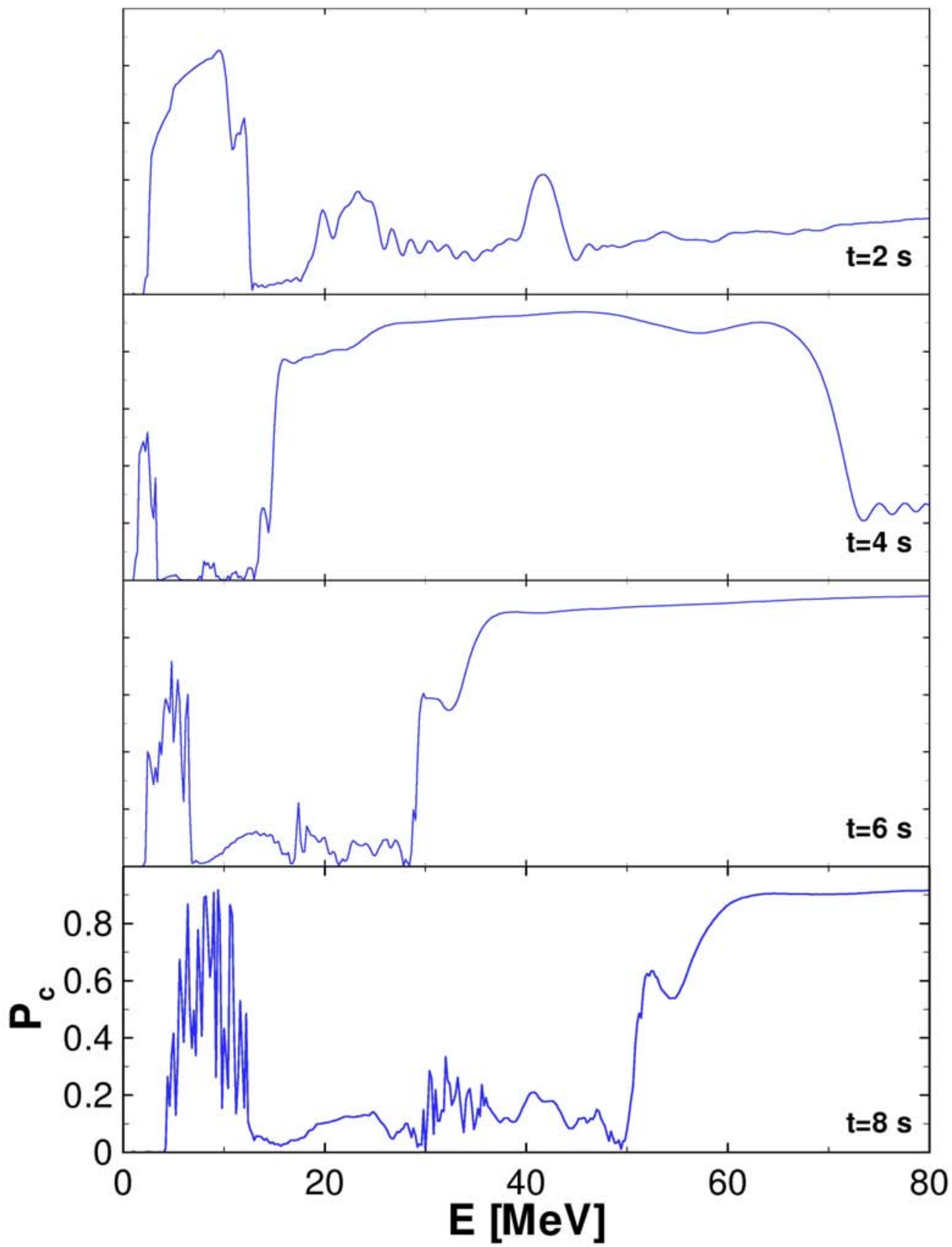
Schirato & Fuller, astro-ph/0205390

Lunardini & Smirnov, JCAP, **6**, 9 (2003)

Likewise, as the bubbles move outward the range of neutrino energies that undergo triple resonances also sweeps through the spectrum.

20° slice,

$\delta m^2 = 3 \times 10^{-3}$ eV^2, $\sin^2(2\theta_V) = 4 \times 10^{-4}$



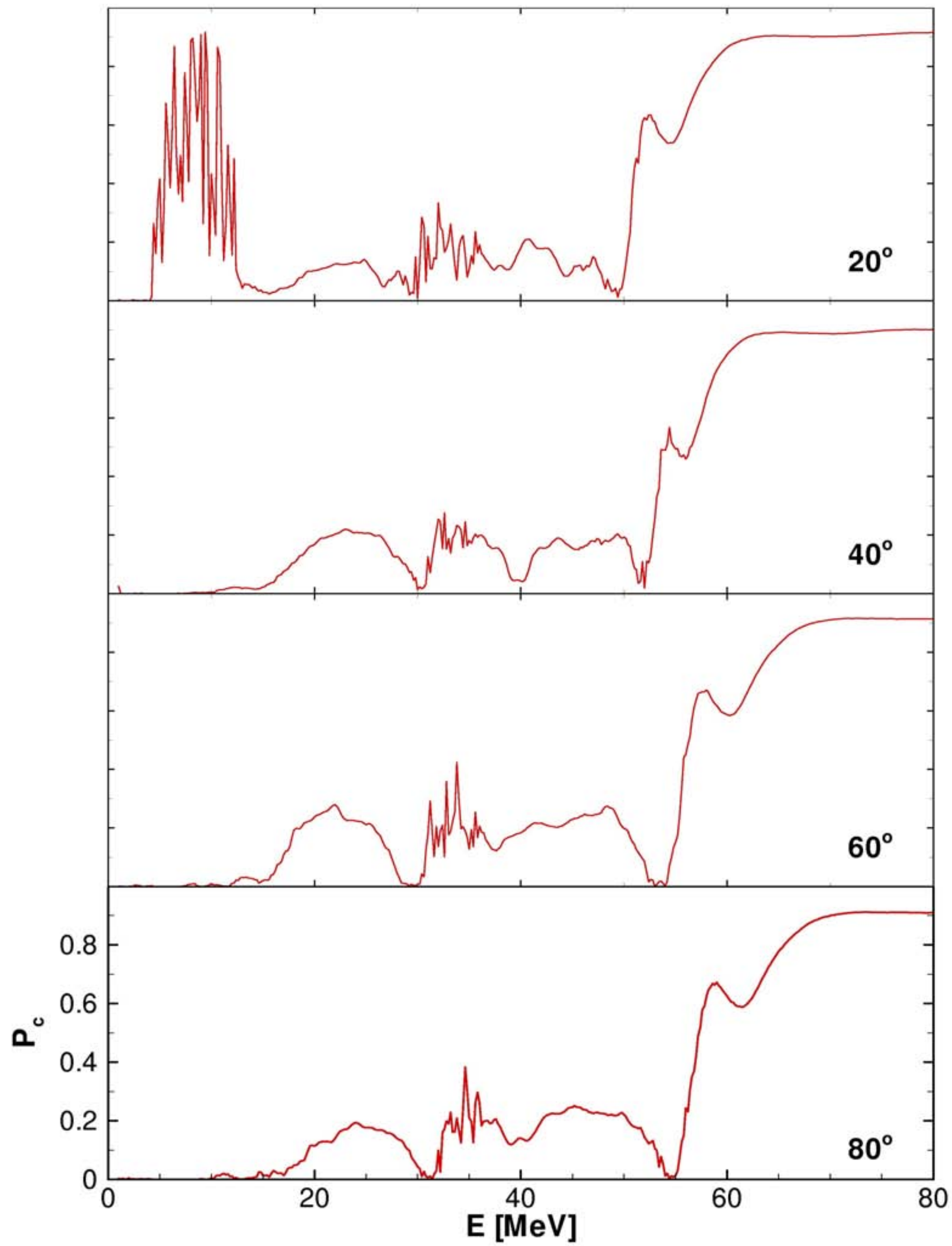
The crossing probability as a function of energy and time possesses evidence of these features.

20° slice,

$\delta m^2 = 3 \times 10^{-3}\ \mathrm{eV}^2$,

$\sin^2(2\theta_V) = 4 \times 10^{-4}$

The crossing probability changes for different radial directions.



$t = 8$ s,

$\delta m^2 = 3 \times 10^{-3}$ eV^2,

$\sin^2(2\theta_V) = 4 \times 10^{-4}$

SN, MSW and MC

The temporal, spatial and energy variations in P_C will modify the fluxes propagating from the proto-neutron star.

The initial flux are taken to be

$$\boxed{F^{(0)} \propto \left(\frac{E}{\langle E \rangle}\right)^{\alpha} \exp\left(-(\alpha+1)\frac{E}{\langle E \rangle}\right)}$$

with $\langle E \rangle$ the mean energy and α controls the 'pinching'.

Keil, Raffelt & Janka, ApJ, **590**, 971 (2003)

$$E_e^{(rms)} = 19\,\mathrm{MeV} \qquad E_{\bar{e}}^{(rms)} = 21\,\mathrm{MeV} \qquad E_{\mu,\tau}^{(rms)} = 23\,\mathrm{MeV}$$

$$L_e = 4.6\times 10^{52}\ \mathrm{ergs/s} \quad L_{\bar{e}} = 4.6\times 10^{52}\ \mathrm{ergs/s} \quad L_{\mu,\tau} = 2.6\times 10^{52}\ \mathrm{ergs/s}$$

Liebendörfer et al, ApJ, **620**, 840 (2005)

The mass splitting and mixing angle we have used are appropriate for 1↔3 mixing at the 'H' resonance.

There is another, 'L', resonance at lower density: we shall work with the assumption that this resonance is always adiabatic.

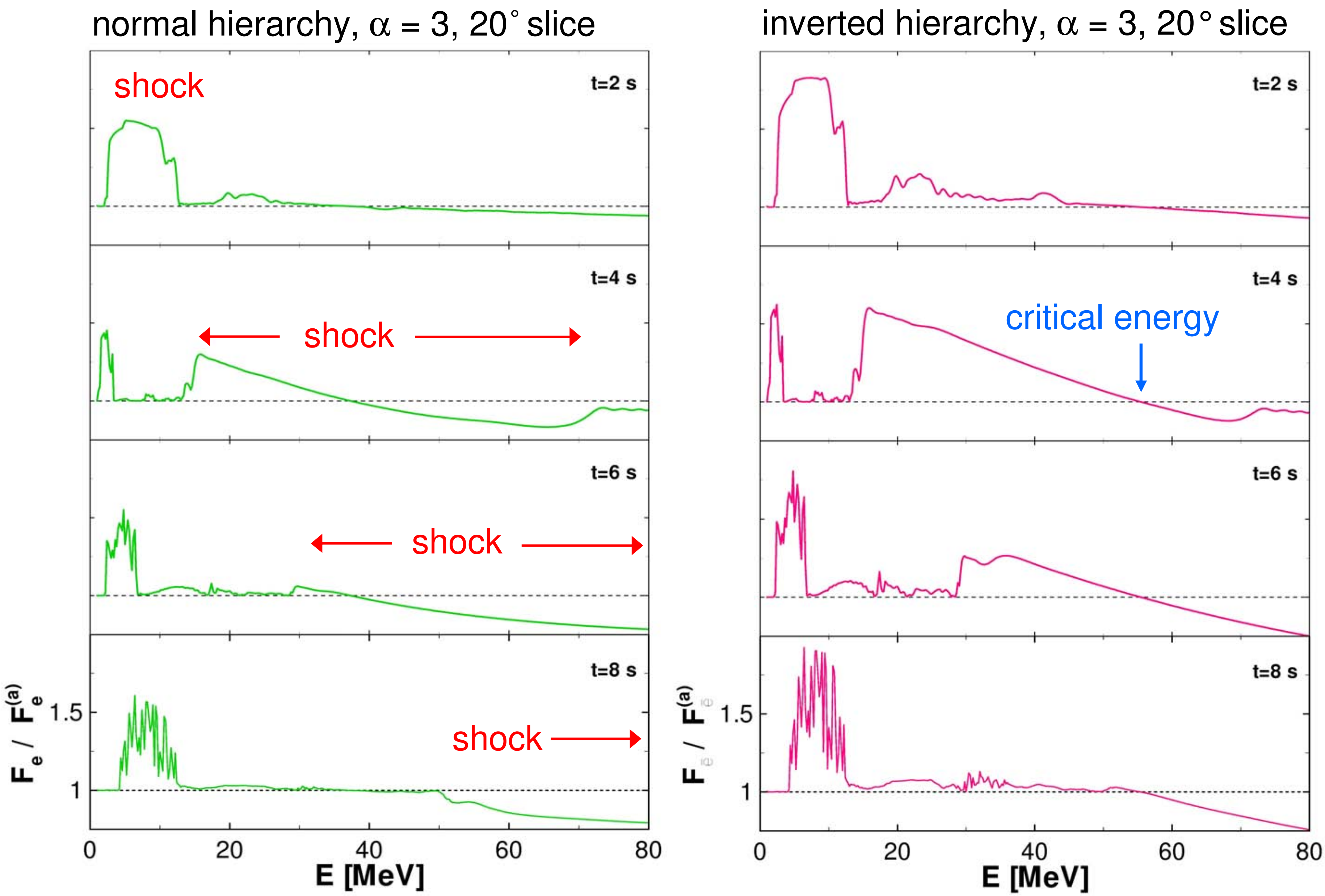
For the mixing angle we used $\sin^2(\theta_{12})=0.28$.

Introducing

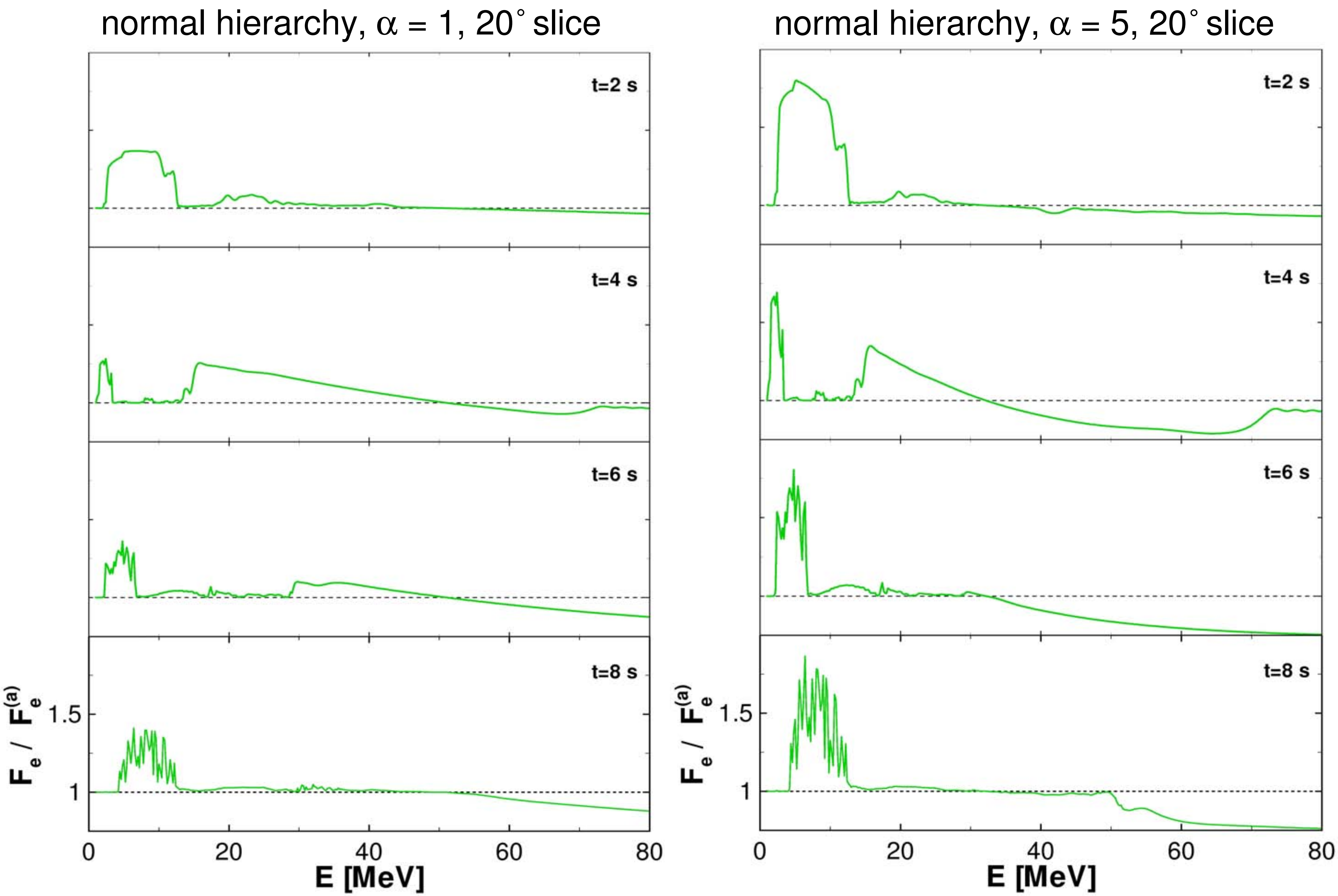
$$\boxed{F_e = p\,F_e^{(0)} + (1-p)\,F_x^{(0)} \qquad F_{\bar{e}} = \bar{p}\,F_{\bar{e}}^{(0)} + (1-\bar{p})\,F_x^{(0)}}$$

then

	$P_{C,H} \sim 0$	$P_{C,H} \sim 1$
Normal	$p = \sin^2\theta_{13}$	$p = \sin^2\theta_{12}$
Inverted	$\bar{p} = \sin^2\theta_{13}$	$\bar{p} = \cos^2\theta_{12}$



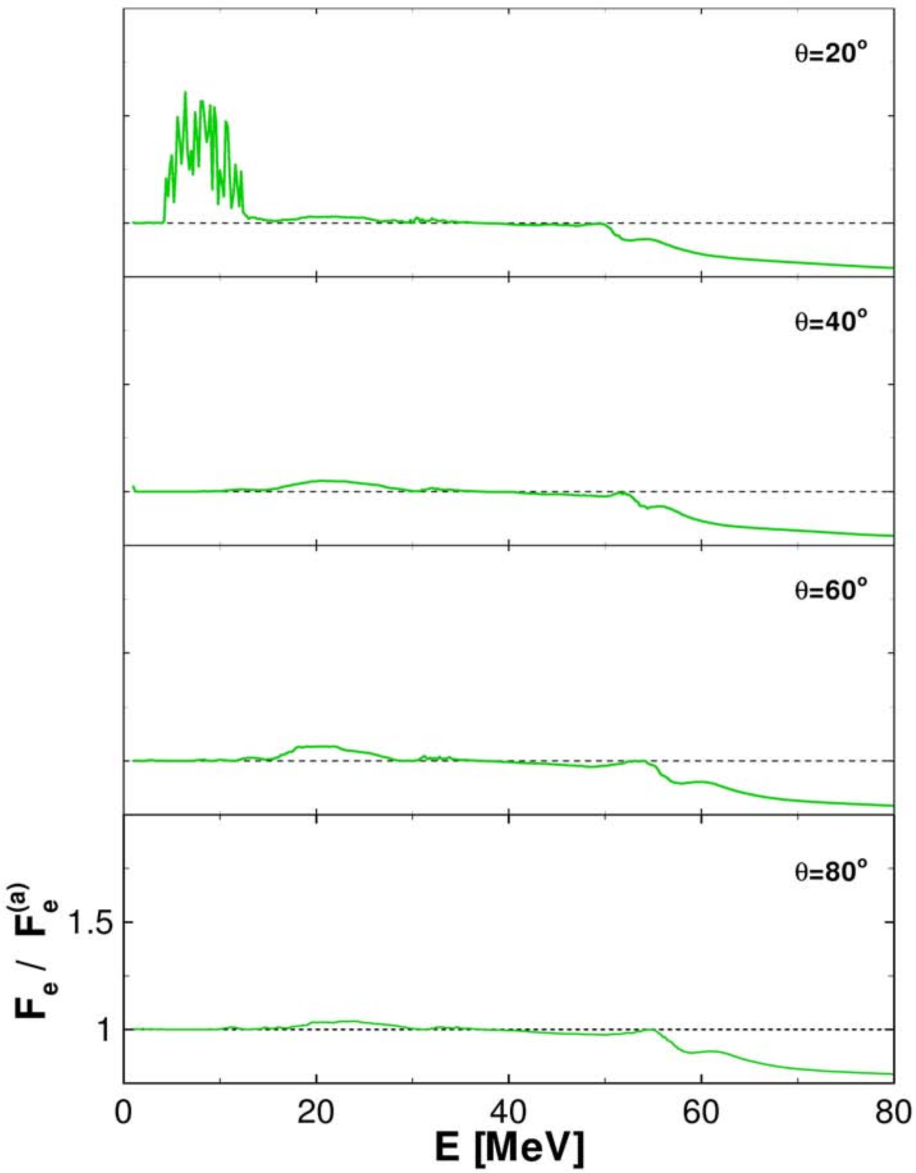
normal hierarchy, α = 3, 20° slice
inverted hierarchy, α = 3, 20° slice
shock
t=2 s
t=4 s
t=6 s
t=8 s
shock
shock
shock
critical energy
$F_e / F_e^{(a)}$
$F_{\bar{e}} / F_{\bar{e}}^{(a)}$
1.5
1
0
20
40
60
80
E [MeV]

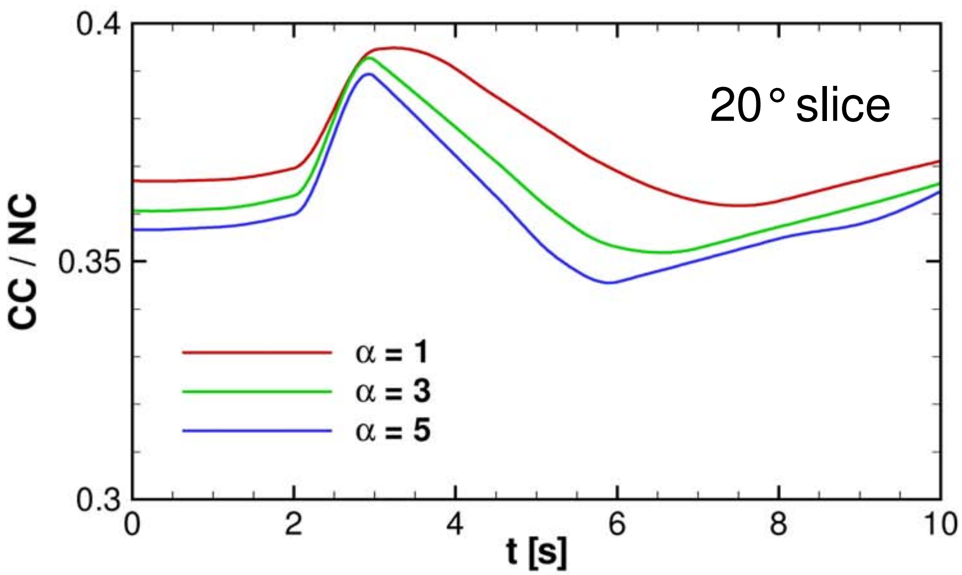


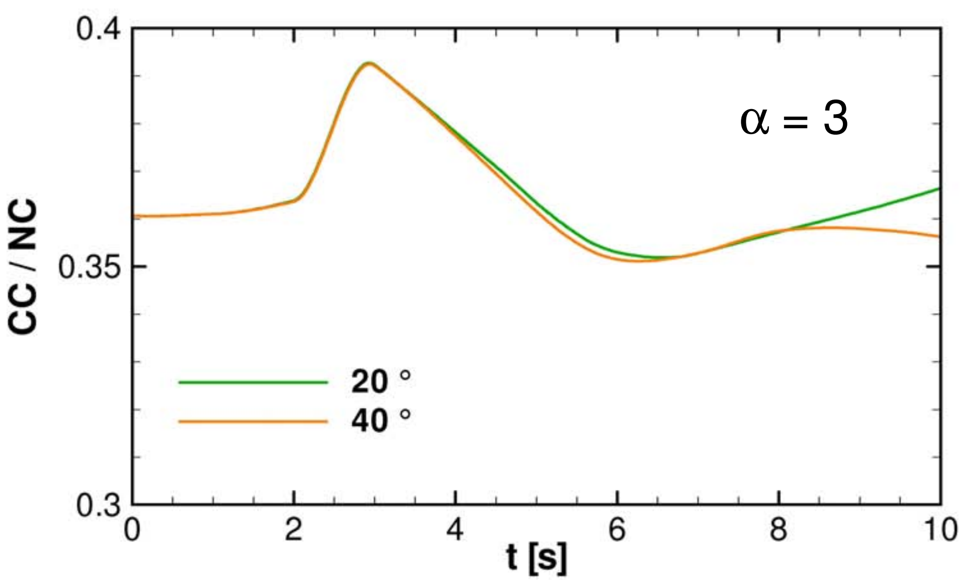
normal hierarchy, α = 1, 20° slice
normal hierarchy, α = 5, 20° slice
t=2 s
t=4 s
t=6 s
t=8 s
t=2 s
t=4 s
t=6 s
t=8 s
$F_e / F_e^{(a)}$
1.5
1
$F_e / F_e^{(a)}$
1.5
1
0
20
40
60
80
E [MeV]
0
20
40
60
80
E [MeV]

The evolution of the neutrino signal will be different for other lines of sight.

normal hierarchy, α = 3, t = 8 s







The ratio of charged current to neutral current events in SNO would largely remove any other time dependence of the flux.

In Summary

- The propagation of a neutrino through a density profile may be recast as a ‘scattering’ of the initial wavefunction.
- MC calculations of S do not suffer from the numerical problems associated with the Schrödinger equation.
- Though less efficient than using an approximation, this method works for all profiles and mixing parameters.
- It should be possible to generalized to 3+ mixing.

- If θ_{13} is not too small then the neutrino spectrum will be modified as the shock passes through the profile.

- This leads to a lower limit on θ_{13}, the hierarchy, and can be used to measure properties of the shock.

- The density profile along the line of sight may not be a monotonically decreasing function of the radius and some neutrinos can experience multiple resonances.

- Such features will also appear in the neutrino signal and allow us to peer inside the exploding star.